Midterm Review Problems

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Exercise 1. A solid conducting sphere of radius a is in a constant, uniform external electric field $\mathbf{E}_0$. It is cut in half into two identical halves with an infinitely thin cut, which is perpendicular to $\mathbf{E}_0$. What force $\mathbf{F}$ acts on each half? How will this force change if we turn off the external field $\mathbf{E}_0$?

Exercise 2. Recall the image solution to a point charge outside a grounded conducting sphere: a charge $q' = -qa/b$ at a distance $b' = a^2/b$ from the center of the sphere, where the charge q is at a distance b from the center of the sphere of radius a. On the homework, you found (in Griffiths 3.9) the solution to this configuration where the sphere is a *neutral* conducting sphere.

a) Find the energy of the configuration in Griffiths 3.9.

b) Find the image solution to a point dipole with dipole moment $\mathbf{p}$ placed at a distance b from the center of a neutral conducting sphere of radius a in the two orientations:

   i) The dipole points in the direction towards the center of the sphere.

   ii) The dipole is perpendicular to the direction in (a).

Exercise 3. The region between two parallel infinite conducting plates at $x = 0$ and $x = L$ is filled with charge of charge density $\rho = \rho_0 \sin(\pi x/L)$. Find the potential and electric field between the plates.

Exercise 4. ***Griffiths 3.55.***

a) A long metal pipe of square cross-section (side a) is grounded on three sides, while the fourth (insulated from the rest) is maintained at constant potential V_0. Show that the net charge per unit length on the side opposite V_0 is

$$\lambda = -\frac{\varepsilon_0 V_0}{\pi} \ln 2.$$

b) A long metal pipe of circular cross-section of radius R is divided lengthwise into four equal sections, three of them grounded and the fourth maintained at constant potential V_0. Show that the net charge per unit length on the section opposite V_0 is the same as in (a).

Exercise 5. ***Griffiths 3.28.*** A charge is distributed with uniform linear charge density λ over the circumference of a circle of radius R which lies in the (x, y)-plane with center at the origin.

a) Find the potential $V(z)$ on the z-axis.

b) Find the first three terms in the multipole expansion for $V(r, \theta)$.

Exercise 6. Six equal by absolute value charges are placed at the vertices of a regular hexagon. The signs of any two neighboring charges are opposite. What kind of multipole does the following system form? By what power law does the potential decay at large distances r from the center of the hexagon?

Exercise 7. The center of a metal sphere of radius a lies on the flat boundary between two dielectric regions of permittivities ε_1 and ε_2. At a distance b from the center of the sphere in the region with permittivity ε_1 is placed a point charge q.

a) Find the potential of the sphere if it is insulated and uncharged.
Hint: If you find the total charge which is induced in the dielectrics, then you can use your solution to Griffiths 3.9 from Homework 3 to determine the potential of the sphere. You should get

$$V = \frac{q}{2\pi b(\varepsilon_1 + \varepsilon_2)}.$$

b) Find the charge induced on the sphere if it is grounded.

Exercise 8. A half-infinite dipole string with linear dipole moment density $\mathbf{p}_l = p_l \hat{x}$ is placed along the negative z-axis.

a) The potential due to a dipole *at the origin* is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p} \cdot \hat{r}}{r^2}.$$

Generalize this to find the potential $V(x, y, z)$ due to the string. This is probably easier if you use cylindrical coordinates.

b) Investigate the behavior of the potential from (a) as you approach the z-axis in the regions $z < 0$ and $z > 0$.