Final Review Problems

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Exercise 1. A charge is distributed with linear charge density λ over the circumference of a circle of radius R which lies in the (x, y)-plane with center at the origin. Find the potential $V(z)$ on the z-axis in the following cases.

a) λ is uniform.

b) $\lambda = C \sin(n\theta)$, where $n \in \mathbb{N}$, C is a constant, and θ is the polar angle.

c) $\lambda = C\theta$.

Exercise 2. ***Griffiths 5.13.*** Suppose you have two infinite, parallel line charges λ a distance d apart, which are moving at a constant speed v. How great would v have to be for the magnetic attraction to balance the electrical repulsion? Calculate the number, and comment on the result.

Exercise 3. ***Griffiths 6.15.*** If $\mathbf{J}_f = \mathbf{0}$ everywhere, the curl of $\mathbf{H}$ vanishes, so we can express $\mathbf{H}$ as the gradient of a scalar potential W,

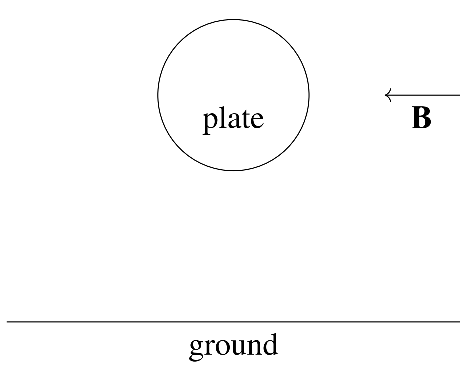
$$\mathbf{H} = -\boldsymbol{\nabla} W.$$

Thus,

$$\nabla^2 W = \boldsymbol{\nabla} \cdot \mathbf{M},$$

so W obeys Poisson's equation with $\boldsymbol{\nabla} \cdot \mathbf{M}$ as the "source." As an example, find the field inside a uniformly magnetized sphere by separation of variables.

Exercise 4. Find the acceleration a of a freely falling, circular, metal plate in a uniform magnetic field which is parallel to the surface of the ground. The plate is oriented with its normal vector (to the circular sides) perpendicular to the direction of the magnetic field and parallel to the ground. The radius of the plate is R and its thickness is $d \ll R$. Its mass is m and the strength of the magnetic field is B.



Exercise 5. A capacitor with circular plates is given an alternating voltage $V(t) = V_0 \sin(\omega t)$. Find the magnetic field within the capacitor if $d \ll a \ll c/\omega$, where d is the distance between the plates and a is the radius of the plates.

Exercise 6. This Exercise is an add-on to Griffiths 7.44-46. Give the change of the self-inductance ΔL of a small circular wire loop if it is placed parallel to a superconducting plane and a distance h above it? Suppose that the radius of the loop is $a \ll h$, and recall that a superconductor can be thought of as a perfect conductor where $\mathbf{B} = \mathbf{0}$ inside.

Exercise 7. Find the energy of the configuration of an uncharged dielectric or metal object which under the effect of a fixed electric field $\mathbf{E}$ obtains a dipole moment $\mathbf{p}$. How will this energy change if the dipole moment $\mathbf{p}$ of the object does not depend on the external field $\mathbf{E}$?

Exercise 8. A plasma can be modeled as a material (usually in gaseous form) in which most of the atoms are ionized. Explain why we should expect it to have an anisotropic dielectric tensor in the presence of an external uniform magnetic field (see Exercise 1 on the Week 15 Worksheet).

Exercise 9. Explain how an AC generator works.

Exercise 10. ***Griffiths 9.39.*** For refraction of light from a medium n_2 into a medium with $n_1 < n_2$, Snell's law has a critical angle

$$\theta_c = \arcsin(n_2/n_1).$$

When the incident angle θ_I is greater than θ_c, there is no refracted ray: We get total internal reflection. However, although no energy penetrates the second medium, there is a nonzero field inside the second medium which is rapidly attenuated. We can use the results from class/the textbook with $k_T = \omega n_2/c$ and

$$\mathbf{k}_T = k_T(\sin\theta_T \hat{x} + \cos\theta_T \hat{z}).$$

However, we should now take

$$\sin\theta_T = \frac{n_1}{n_2}\sin\theta_I > 1,$$

so that

$$\cos\theta_T = i\sqrt{\sin^2\theta_T - 1}$$

is imaginary.

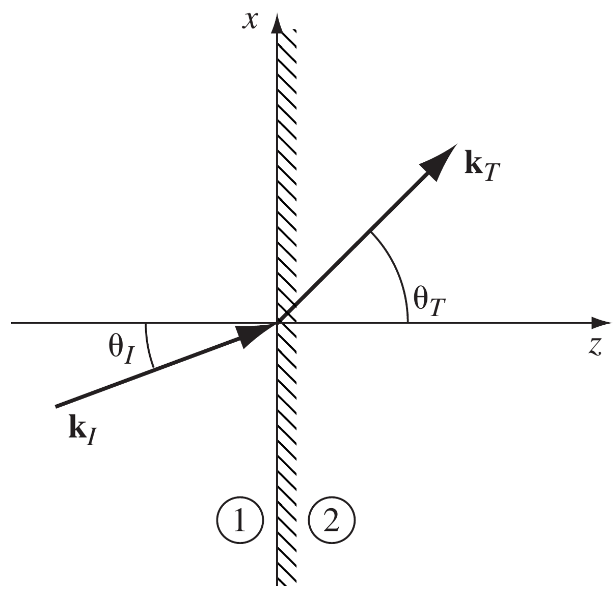
a) Show that

$$\tilde{\mathbf{E}}_T(\mathbf{r}, t) = \tilde{\mathbf{E}}_{0T} e^{-\kappa z} e^{-i(kx - \omega t)},$$

where

$$\kappa = \frac{\omega}{c}\sqrt{(n_1 \sin\theta_I)^2 - n_2{}^2} \quad \text{and} \quad k = \frac{\omega n_1}{c}\sin\theta_I.$$

Notice that this is a wave propagating in the x direction and attenuated in the z direction.



b) Noting that

$$\alpha = \frac{\cos\theta_T}{\cos\theta_I}$$

is now imaginary, use Fresnel's equations

$$\tilde{E}_{0R} = \frac{\alpha - \beta}{\alpha + \beta}\tilde{E}_{0I}$$
$$\tilde{E}_{0T} = \frac{2}{\alpha + \beta}\tilde{E}_{0I}$$

to calculate the reflection coefficient for polarization parallel to the plane of incidence.

c) Do the same for polarization perpendicular to the plane of incidence.

d) In the case of polarization perpendicular to the plane of incidence, show that the real evanescent fields are

$$\mathbf{E}(\mathbf{r},t) = E_0 e^{-\kappa z}\cos(kx - \omega t)\hat{y}$$
$$\mathbf{B}(\mathbf{r},t) = \frac{E_0}{\omega} e^{-\kappa z}\left[\kappa \sin(kx - \omega t)\hat{x} + k\cos(kx - \omega t)\hat{z}\right].$$

e) Check that the fields in (d) satisfy Maxwell's equations.

f) For the fields in (d), construct the Poynting vector, and show that, on average, no energy is transmitted in the z direction.

Exercise 11. The Dirac Monopole. Consider a half-infinite string of magnetic dipoles, equivalently, a half-infinite solenoid, denoted L.

a) Show that the vector potential outside the string is

$$\mathbf{A}(\mathbf{r}) = -\frac{g}{4\pi}\int_L \mathrm{d}\boldsymbol{\ell} \times \nabla\left(\frac{1}{\mathscr{r}}\right),$$

where g is a constant.

b) Show that the curl of **A** is directed radially outward from the end of the string, varies inversely with distance squared from the end of the string, and has total outward flux g.

Remark. The result of (b) shows that the magnetic field outside of the solenoid is that given by a magnetic monopole of exactly charge g. On the other hand, it can be shown (try for yourself!) that changing the position of the string changes **A** by a gauge transformation. Explicitly, if we have two different strings L, L', then the integral taken along the closed path $C = L - L'$ will give

$$\mathbf{A}_{L'}(\mathbf{r}) = \mathbf{A}_L(\mathbf{r}) + \frac{g}{4\pi}\nabla\Omega_C(\mathbf{r}),$$

where Ω_C is the solid angle subtended by the contour C at the observation point $\mathbf{r}$. This means that the string itself is not observable, which is consistent with the fact that physical effects due to the monopole should not depend on the theoretical artifice used to create it (the string). In 1930, Dirac famously showed that the existence of magnetic monopoles *implies* the quantization of electric (and magnetic) charge! This is why people have been interested in magnetic monopoles to this day.