Week 13 Worksheet Solutions
More Electrodynamics

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Exercise 1. A capacitor C is charged up to a voltage V and connected to an inductor L in series at time $t = 0$.

a) ***Griffiths 7.27.*** Find the current in the circuit as a function of time.

b) Show that the total energy of the configuration is constant at any time t, and find this constant.

a) Since the induced emf is

$$\mathcal{E} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

and

$$C = \frac{Q}{V},$$

we have that

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = -\frac{I}{LC}.$$

It follows that a general solution for $I(t)$ is given by

$$I(t) = A\sin(\omega t) + B\cos(\omega t),$$

where $\omega = 1/\sqrt{LC}$. Since $I(0) = 0$, we should set $B = 0$. Now, at $t = 0$, we have

$$\begin{aligned} V_0 &= -L\frac{\mathrm{d}I}{\mathrm{d}t}\bigg|_{t=0} \\ &= -LA\omega. \end{aligned}$$

Hence,

$$A = -\frac{V_0}{L\omega}.$$

The solution for I is then

$$I(t) = -V_0\sqrt{\frac{C}{L}}\sin\left(\frac{t}{\sqrt{LC}}\right).$$

b) The energy stored in the inductor is $\frac{1}{2}LI^2$. Similarly, the energy stored in the capacitor is $\frac{1}{2}CV^2$. We already know I, so we need to figure out V. We can do this using $CV = Q$, which implies

$$C\frac{\mathrm{d}V}{\mathrm{d}t} = I.$$

Hence,

$$V(t) = V_0 \cos\left(\frac{t}{\sqrt{LC}}\right),$$

where we note that the integration constant is 0 since $V(0) = V_0$. Now, we can compute

$$\frac{1}{2}LI^2 + \frac{1}{2}CV^2 = \frac{1}{2}C{V_0}^2.$$