# Week 14 Worksheet
# Waves and Energy

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**Exercise 1.**

a) Starting with Maxwell's equations in vacuum, show that they give you wave equations for $\mathbf{E}$ and $\mathbf{B}$.

b) Show that the waves are transverse: If they travel in the $z$ direction, then $\widetilde{E}_{0z} = 0$ and $\widetilde{B}_{0z} = 0$. *Hints*: If the waves travel in the $z$-direction, what is the spatial dependence of $\widetilde{E}$ and $\widetilde{B}$? Once you determine this, use the equations $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$.

c) Show that (for waves traveling in the $z$-direction)

$$\mathbf{B} \propto \hat{z} \times \mathbf{E},$$

so that $\mathbf{B}$ is perpendicular to and in phase with $\mathbf{E}$.

**Exercise 2.** Recalling that magnetic forces do no work, we have that

$$dW = \mathbf{F} \cdot d\boldsymbol{\ell} = q\mathbf{E} \cdot \mathbf{v}\,dt.$$

Thus, the rate at which work is done on the charges in a volume is

$$\frac{dW}{dt} = \int \mathbf{E} \cdot \mathbf{J}\,dV.$$

a) Show that

$$\mathbf{E} \cdot \mathbf{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\nabla \cdot \mathbf{E} \times \mathbf{B}$$

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*Hints*: Use Maxwell's equations along with the identity

$$\nabla \cdot (\mathbf{v} \times \mathbf{w}) = \mathbf{w} \cdot \nabla \times \mathbf{v} - \mathbf{v} \cdot \nabla \times \mathbf{w}.$$

b) Plug this in to the formula for the rate of work, and derive Poynting's theorem:

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$$

is the energy per unit time, per unit area, transported by the fields; it is the energy flux density.

c) You showed in class that

$$\mathbf{g} = \mu_0 \varepsilon_0 \mathbf{S}$$
$$= \frac{1}{c^2} \mathbf{S}$$

is the momentum per unit volume stored in the fields. Discuss why $\mathbf{S}$ seems to have two different physical interpretations.