Week 15 Worksheet
EM Waves in Matter

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Exercise 1. A plasma in a uniform magnetic field $\mathbf{B} = B\hat{z}$ is a **gyrotropic material**, which means it has an anisotropic dielectric tensor $\varepsilon = (\varepsilon_{ij})$ with nonzero components $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_1$, $\varepsilon_{12} = -\varepsilon_{21} = i\varepsilon_2$, $\varepsilon_{33} = \varepsilon_3$, where the ε_i are real numbers (in the absence of losses). Assume that the plasma has $\mu = \mu_0$ and that there are no free charges or currents.

a) Show that the equation $\mathbf{D} = \varepsilon\mathbf{E}$ for such a material can be written in the form
$$\mathbf{D} = \varepsilon_\parallel E_z\hat{z} + \varepsilon_\perp\mathbf{E}_\perp + i\mathbf{E}\times\mathbf{g},$$
where $\mathbf{g} = g\hat{z}$ is the **gyration vector** and $\mathbf{E}_\perp$ is the part of $\mathbf{E}$ perpendicular to $\mathbf{g}$. Make sure you determine $\varepsilon_\parallel$, $\varepsilon_\perp$, and g in terms of the components of ε.

b) Look for monochromatic plane wave solutions to Maxwell's equations with wave vector $\mathbf{k} = k\hat{z}$. Show that Maxwell's equations reduce to a single eigenvalue-eigenvector equation for $\mathbf{E}$.
Hint: Show first that $E_z = 0$, so that $\mathbf{E} = \mathbf{E}_\perp$.

c) Solve the equation from (b), and determine the allowed magnitudes of the wavevector $k_\pm$.

d) Determine the type of polarization (in the sense of Exercise 2, below) of your solutions to (b) and (c).

Exercise 2. Polarization. When we solve Maxwell's equations in vacuum, we end up with monochromatic plane wave solutions (traveling in the z-direction) of the form
$$\mathbf{E} = (E_1\hat{x} + E_2\hat{y})e^{i(kz-\omega t)},$$
where the E_i are in general complex numbers. If the phases of E_i are *the same*, we say that the wave is **linearly polarized**. If the phases of E_i differ by $\pi/2$ but the magnitudes $|E_1| = |E_2| = E_0$ are the same, we say the wave is **circularly polarized**.

a) Explain why the name "linear polarization" makes sense.

b) For linearly polarized $\mathbf{E}$, what is $\mathbf{B}$? Is it also linearly polarized?

c) Do the same for "circular polarization." A wave proportional to $\hat{x} + i\hat{y}$ is said to have **negative helicity** or be **left circularly polarized**, while a wave proportional to $\hat{x} - i\hat{y}$ is said to have **positive helicity** or be **right circularly polarized**.

d) **Challenge:** What would you call "elliptical polarization?" Why?
Hint: Write $\mathbf{E} = E_0(e^{i\delta/2}\hat{x} + e^{-i\delta/2}\hat{y})e^{i(kz-\omega t)}$.