Week 6 Worksheet
Polarization and Dielectrics

Jacob Erlikhman

February 23, 2026

Exercise 1.

a) Consider a line of dipoles of equal moment qd, where the end of one touches the beginning of the next. Suppose the line is of length ℓ. What is the dipole moment of the entire line?

b) Now, suppose you had a cylinder of length ℓ, cross-sectional area A, and uniform polarizaiton $\mathbf{P}$, which is parallel to the axis of the cylinder. What is the surface charge density on the ends of the cylinder?

c) Argue that the bound surface charge for any uniformly polarized object is

$$\sigma_b = \mathbf{P} \cdot \hat{n}.$$

Hint: Consider the cylinder as in (b), with axis along the x-axis, but this time with one of the ends cut at an angle θ relative to the y-axis.

d) What happens if the polarization is *nonuniform*? Consider a sphere with a diverging $\mathbf{P}$. The charge which accumulates at the edges must be σ_b, so what is the net charge inside the sphere?

e) Argue that

$$\int \rho_b \, \mathrm{d}V = -\oint \mathbf{P} \cdot \mathrm{d}\mathbf{a},$$

and use the divergence theorem to obtain that $\rho_b = -\nabla \cdot \mathbf{P}$ for the sphere of (d). Argue that this holds for any object.

Exercise 2. A dielectric of arbitrary shape, volume V, and relative permittivity ε_r which is close to 1 (i.e. such that $\varepsilon_r - 1 \ll 1$) is brought into a uniform electric field $\mathbf{E}$. Outside the dielectric, $\varepsilon_r = 1$. Find the field at a large distance r from the dielectric.
Hint: By taking the gradient in spherical coordinates of the dipole potential V_{dip}, one can obtain that the field due to a dipole $\mathbf{p} = p\hat{z}$ at the origin is

$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}).$$