# Midterm Review Problems

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**Exercise 1.** A solid conducting sphere of radius $a$ is in a constant, uniform external electric field $\mathbf{E}_0$. It is cut in half into two identical halves with an infinitely thin cut, which is perpendicular to $\mathbf{E}_0$. What force $\mathbf{F}$ acts on each half? How will this force change if we turn off the external field $\mathbf{E}_0$?

**Exercise 2.** Recall the image solution to a point charge outside a grounded conducting sphere: a charge $q' = -qa/b$ at a distance $b' = a^2/b$ from the center of the sphere, where the charge $q$ is at a distance $b$ from the center of the sphere of radius $a$.

a) ***Griffiths 3.9.*** Find the image solution to the above configuration where the sphere is a *neutral* conducting sphere. Also find the force on the charge and the energy of the configuration.

b) Find the image solution to a point dipole with dipole moment $\mathbf{p}$ placed at a distance $b$ from the center of a neutral conducting sphere of radius $a$ in the two orientations: 1) The dipole points in the direction towards the center of the sphere; 2) the dipole is perpendicular to the previous direction.

**Exercise 3.** The region between two parallel infinite conducting plates at $x = 0$ and $x = L$ is filled with charge of charge density $\rho = \rho_0 \sin(\pi x/L)$. Find the potential and electric field between the plates.

**Exercise 4.** ***Griffiths 3.55.*** a) A long metal pipe of square cross-section (side $a$) is grounded on three sides, while the fourth (insulated from the rest) is maintained at constant potential $V_0$. Show that the net charge per unit length on the side opposite $V_0$ is

$$\lambda = -\frac{\varepsilon_0 V_0}{\pi} \ln 2.$$

b) A long metal pipe of circular cross-section of radius $R$ is divided lengthwise into four equal sections, three of them grounded and the fourth maintained at constant potential $V_0$. Show that the net charge per unit length on the section opposite $V_0$ is the same as in (a).

**Exercise 5.** ***Griffiths 3.28.*** A charge is distributed with uniform linear charge density $\lambda$ over the circumference of a circle of radius $R$ which lies in the $(x, y)$-plane with center at the origin.

a) Find the potential $V(z)$ on the $z$-axis.

b) Find the first three terms in the multipole expansion for $V(r, \theta)$.

**Exercise 6.** Six equal by absolute value charges are placed at the vertices of a regular hexagon. The signs of any two neighboring charges are opposite. What kind of multipole does the following system form? By what power law does the potential decay at large distances $r$ from the center of the hexagon?