Week 10 Worksheet
Polarization and Dielectrics

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Exercise 1. a) Consider a line of dipoles of equal moment qd, where the end of one touches the beginning of the next. Suppose the line is of length ℓ. What is the dipole moment of the entire line?

b) Now, suppose you had a cylinder of length ℓ, cross-sectional area A, and uniform polarizaiton $\mathbf{P}$, which is parallel to the axis of the cylinder. What is the surface charge density on the ends of the cylinder?

c) Argue that the bound surface charge for any uniformly polarized object is

$$\sigma_b = \mathbf{P} \cdot \hat{n}.$$

Hint: Consider the cylinder as in (b), with axis along the x-axis, but this time with one of the ends cut at an angle θ relative to the y-axis.

d) What happens if the polarization is *nonuniform*? Consider a sphere with a diverging $\mathbf{P}$. The charge which accumulates at the edges must be σ_b, so what is the net charge inside the sphere?

e) Argue that

$$\int \rho_b \, \mathrm{d}V = -\oint \mathbf{P} \cdot \mathrm{d}\mathbf{a},$$

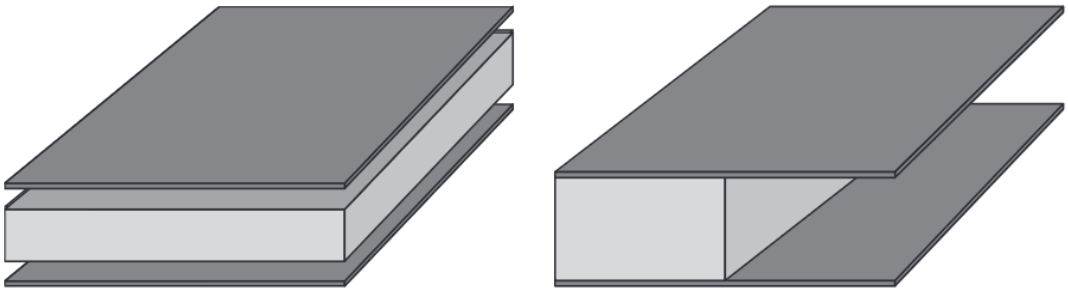
and use the divergence theorem to obtain that $\rho_b = -\boldsymbol{\nabla} \cdot \mathbf{P}$ for the sphere of (d). Argue that this holds for any object.

Exercise 2. A linear dilectric of susceptibility χ_e is one in which

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}.$$

What is the relation between $\mathbf{D}$ and $\mathbf{E}$ for such dielectrics, and what is the "permittivity," ε, of the material in terms of χ_e?

Exercise 3. ***Griffiths 4.19.*** Suppose you have enough linear dielectric matrial of dielectric constant ε_r to half-fill a parallel-plate capacitor. By what fraction is the capacitance increased when you distribute the material as in (a) vs. as in (b) in the figure below? For a fixed potential difference V between the plates, find $\mathbf{E}, \mathbf{D}$, and $\mathbf{P}$ in each region and the free and bound charge on all surfaces in both cases.
Hint: For the second part of the exercise, use $C = Q/V$ to find Q; then, use that to find everything else. Why can't we just use $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$ to find the polarization and hence everything else?



(a) (b)