Week 12 Worksheet
Magnets!

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Exercise 1. Suppose a magnetized object has a *uniform* magnetization $\mathbf{M}$.

a) What does $\int \mathbf{M}\,\mathrm{d}V$ calculate, where the integral is taken over the volume of the object?

b) Suppose a little current loop creates one of the infinitesimal dipole moments. Let it have area a and thickness t. What is m for this loop?

c) Since $m = Ia$, find an equation for the current I, and hence for the bound surface current K_b.
Hint: You can write K_b in terms of I and the dimension(s) of the loop; then, you can eliminate the dimensions using the previous results.

d) Deduce that

$$\mathbf{K}_b = \mathbf{M} \times \hat{n}.$$

Exercise 2. Suppose a magnetized object has a *non-uniform* magnetization $\mathbf{M}$. We already know how to calculate the surface current density, so we would now like to calculate the volume current density.

a) Consider two adjacent infinitesimal chunks of magnetized material, one at (x, y, z) and the other at $(x, y + \mathrm{d}y, z)$. On the surface where the chunks touch, which way does the net current point? Show that its magnitude is

$$\frac{\partial M_z}{\partial y}\,\mathrm{d}y\,\mathrm{d}z.$$

b) What is the net bound current density due the touching surface for the chunks considered in (b)?

c) Now, consider two chunks situated at (x, y, z) and $(x, y, z + \mathrm{d}z)$, and consider M_y. Obtain that the net bound current density due to the chunks in (c) and the new chunks is

$$(J_b)_x = (\nabla \times \mathbf{M})_x.$$

Exercise 3. We can write $\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$. Since $\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J}$, write $\mathbf{J}_b = \nabla \times \mathbf{M}$ to obtain a definition for $\mathbf{H}$, where

$$\nabla \times \mathbf{H} = \mathbf{J}_f.$$

Exercise 3. ***Griffiths 6.13*.** Suppose the field inside a large piece of magnetic material is $\mathbf{B}_0$, so that $\mathbf{H}_0 = \frac{1}{\mu_0}\mathbf{B}_0 - \mathbf{M}$, where $\mathbf{M}$ is a "frozen-in" magnetization. Assume the cavities are small enough so that $\mathbf{M}$, $\mathbf{B}_0$, and $\mathbf{H}$ are essentially constant inside them.
Hint: What is the magnetization inside the cavity? Is there a way you could induce that magnetization by using a different configuration and the property of superposition?

a) Now, a small spherical cavity is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{B}_0$ and $\mathbf{M}$. Also find $\mathbf{H}$ at the center of the cavity in terms of $\mathbf{H}_0$ and $\mathbf{M}$.

b) Do the same for a long needle-shaped cavity running parallel to $\mathbf{M}$.

c) Do the same for a thin wafer-shaped cavity perpendicular to $\mathbf{M}$. (A wafer is a disk with a small thickness relative to its radius).