Week 4 Worksheet
Electrostatics

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Exercise 1. Using Dirac delta functions in the appropriate coordinates if necessary, express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{r})$.

a) In spherical coordinates, a charge Q uniformly distributed over a spherical shell of radius R.

b) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b.

c) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.

Exercise 2. Two infinite parallel plates carry equal and opposite uniform charge densities $\pm\sigma$. Put the positively charged plate in the (x, y)-plane and the negatively charged one at $z = 1$ above it. Find the electric field in each of three regions: $z < 0$, $0 < z < 1$, and $z > 1$.

Exercise 3. a) The potential at a point $\mathbf{r}$ is defined as

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\boldsymbol{\ell},$$

where $\mathcal{O}$ is some reference point. Explain why this is well-defined (i.e. unambiguous, up to the choice of $\mathcal{O}$).
Hint: Use Stokes' theorem.

b) An infinite plate carries a uniform charge density σ. Using your result from Exercise 2, find the potential everywhere.
Hint: Where would you put your reference point $\mathcal{O}$?

Exercise 4. Consider a uniformly charged spherical shell of radius R and charge Q.

a) Find the electric field everywhere using Gauss' law.

b) Find the potential everywhere by direct integration, i.e. using

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{\mathscr{r}} \, d\tau',$$

where the integral is taken over the shell.
Hints: Consider a single point a distance z from the center of the sphere, and use *cylindrical* symmetry (but spherical coordinates). Also, you can figure out what $\mathscr{r}$ is using the cosine law. Lastly, **be very careful to take the positive square root: Consider the separate cases $z < R$ and $z > R$.**

c) Set up the integral to find the electric field at a point a distance z from the center of the sphere (without using Gauss' law). Compute all the integrals except the θ' integral, so that your final answer is of the form

$$\mathbf{E} = \int \text{stuff}\, \mathrm{d}\cos\theta' \hat{r}$$

(an integral over θ' is fine too).
Hint: The $\hat{\mathscr{r}}$ that appears in the original integral does not point in $\hat{r}$, but $\mathbf{E}$ does. What happens to the electric field in the non-radial directions?

Exercise 5. Repeat Exercise 4, but this time for a solid sphere of radius R and charge Q. You can use the results of Exercise 4 in your answers (i.e. you don't need to repeat integrals you did/set up in that problem).