Week 5 Worksheet
More Electrostatics

Jacob Erlikhman

Exercise 1. a) The potential at a point $\mathbf{r}$ is defined as

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot \mathrm{d}\boldsymbol{\ell},$$

where $\mathcal{O}$ is some reference point. Explain why this is well-defined (i.e. unambiguous, up to the choice of $\mathcal{O}$).
Hint: Use Stokes' theorem.

b) An infinite plate carries a uniform charge density σ. Find the potential everywhere.
Hints: Recall that the electric field is $E = \sigma/2\varepsilon_0$. Where would you put your reference point $\mathcal{O}$?

Exercise 2. Consider a uniformly charged spherical shell of radius R and charge Q.

a) Find the electric field everywhere using Gauss' law.

b) Find the potential everywhere by direct integration, i.e. using

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\mathbf{r}')}{\scriptstyle \mathcal{R}} \,\mathrm{d}\tau',$$

where the integral is taken over the shell.
Hints: Consider a single point a distance z from the center of the sphere, and use *cylindrical* symmetry (but spherical coordinates). Also, you can figure out what $\mathcal{R}$ is using the cosine law. Lastly, **be very careful to take the positive square root: Consider the separate cases $z < R$ and $z > R$.**

c) Set up the integral to find the electric field at a point a distance z from the center of the sphere (without using Gauss' law). Compute all the integrals except the θ' integral, so that your final answer is of the form

$$\mathbf{E} = \int \text{stuff} \,\mathrm{d}\cos\theta' \hat{r}$$

(an integral over θ' is fine too).
Hint: The $\hat{\mathcal{R}}$ that appears in the original integral does not point in $\hat{r}$, but $\mathbf{E}$ does. What happens to the electric field in the non-radial directions?