Week 11 Worksheet

Jacob Erlikhman

November 3, 2026

Exercise 1 (3D Harmonic Oscillator)**.** Consider 3-dimensional quantum mechanics with the potential $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$.

a) Show that $\psi = X(x)Y(y)Z(z)$ separates the problem into three 1D oscillators and give $E_{n_x n_y n_z}$ in terms of $N \equiv n_x + n_y + n_z$.

b) Give the energies and degeneracies of the three lowest levels, and a formula for the degeneracy $d(N)$.

c) Now make the oscillator *anisotropic*, $V = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$. Give the energies and compute $[\hat{H}, \hat{L}_z]$. For which frequencies does it vanish?

d) For $\omega_x = \omega_y = \frac{2}{3}\omega_z$, give the three lowest levels and their degeneracies.

Exercise 2 (Infinite Spherical Well)**.** A particle of mass m is free inside a sphere of radius a and cannot escape. Use the radial equation from lecture,

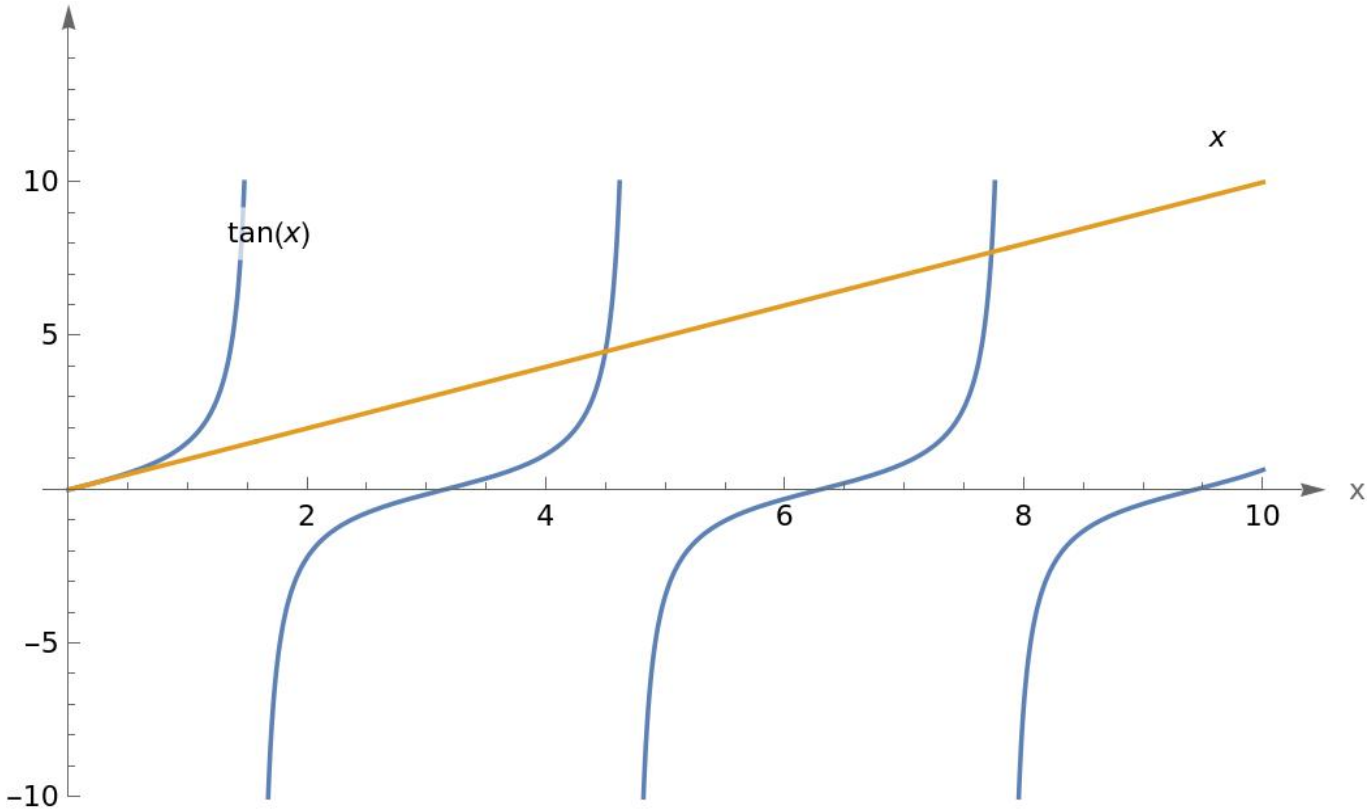
$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{\hbar^2}{2M}\frac{\ell(\ell+1)}{r^2}\right]u = Eu, \qquad u(r) = rR,$$

where $\int_0^\infty |u|^2 dr = 1$.

a) Set $\ell = 0$, write the equation and both boundary conditions, and name the 1D problem it coincides with.

b) Find the energies E_n and the normalized $R_n(r)$.

c) For $\ell = 1$ the regular solution is $u \propto r\, j_1(kr)$ with $j_1(x) = \sin(x)/x^2 - \cos(x)/x$. Find the first two lowest $\ell = 1$ energies (graphically—see the back of the worksheet).


x
10
tan(x)
5
2
4
6
8
10
x
−5
−10