Week 13 Worksheet

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Exercise 1. For the ground state of the hydrogen atom, calculate the probability that the electron and the proton are farther apart than would be permitted by classical mechanics at the same total energy. Recall that the ground state is given by

$$\begin{aligned}\psi(r,\theta,\varphi) =& R_{10}(r)Y_{00}(\theta,\varphi)\\ =& Ae^{-r/a},\end{aligned}$$

where a is the Bohr radius and A is a normalization constant.
Hint: Recall that the ground state energy is $E_1 = -me^4/2\hbar^2$.

Exercise 2. A particle has $\psi(\mathbf{x}) = \left(\sqrt{2}\,x + \sqrt{2}\,y + z\right) f(r)$ with f spherically symmetric.

a) Show that

$$\frac{z}{r} = \sqrt{\frac{4\pi}{3}}\,Y_1^0, \qquad \frac{x \pm iy}{r} = \mp\sqrt{\frac{8\pi}{3}}\,Y_1^{\pm 1}.$$

b) Is ψ an eigenfunction of L^2? If so, give the eigenvalue.

c) Give the probabilities of finding $m_\ell = -1$, 0 and 1.

d) Suppose ψ is an energy eigenfunction with eigenvalue E and $f(r) = 3r^2$. Find $V(r)$.