Midterm 1 Review

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July 8, 2026

Exercise 1. Show that if $E < V(x)$ for all x, then $\psi(x) \equiv 0$.

Exercise 2. Suppose a free particle moves not on the x-axis but on a circle of radius R.

a) Find the stationary states and the allowed energies.

b) You should have found in (a) that there are degenerate states (i.e. linearly independent stationary states with the same energy). How is that consistent with the result of Exercise 2 of the second week 2 discussion section worksheet?

Exercise 3. Calculate the momentum space wavefunctions for the harmonic oscillator, $\varphi_n(p)$.
Hint: Use the momentum space differential equation $H\varphi = E\varphi$ and that $x = i\hbar d/dp$.

Exercise 4. ***Griffiths 2.43.*** Solve the TISE for a centered infinite square well with a δ-funciton barrier in the middle:

$$V(x) = \begin{cases} \alpha\delta(x), -a < x < a \\ \infty, |x| \geq a \end{cases}.$$

Don't bother to normalize your solutions, but find the allowed energies. Comment how the energies compare with the corresponding energies in the absence of the δ-function. Explain why the odd solutions are not affected by the δ-function. Comment on the limiting cases $\alpha \to 0$ and $\alpha \to \infty$.

Exercise 5. ***Griffiths 2.27-8.*** Consider the double δ-function potential

$$V(x) = -\alpha[\delta(x + a) + \delta(x - a)],$$

where $\alpha, a > 0$.

a) Sketch this potential.

b) How many bound states does it possess? Find the allowed energies for $\alpha = \hbar^2/ma$ and $\alpha = \hbar^2/4ma$, and sketch the wave functions.

c) What are the bound state energies in the limit cases (i) $a \to 0$ and (ii) $a \to \infty$ (holding α fixed)? Explain why your answers are reasonable.

d) Find the transmission coefficient.

Exercise 6. ***Griffiths 2.33.***

a) Find the transmission coefficient for a rectangular barrier; that is, a potential with $V(x) = V_0 > 0$ in the region $x \in (-a, a)$. Treat separately the three cases $E < V_0$, $E = V_0$, and $E > V_0$.

b) Explain in what sense a plane wave solution $\psi(x) = Ae^{ikx}$ is a "wave traveling to the right." Same thing for plane waves $\psi(x) = Ae^{-ikx}$ as a "wave traveling to the left."

Exercise 7. ***Griffiths 2.47*** Consider a double finite square well:

$$V(x) = \begin{cases} -V_0, & x \in (-a - b/2, b/2) \\ -V_0, & x \in (b/2, a + b/2) \\ 0, & \text{otherwise} \end{cases} .$$

a) Sketch this potential.

b) Sketch the allowed wave functions for the cases (i) $b = 0$, (ii) $b \approx a$, and (iii) $b \gg a$. Include also the ground state and first excited state.

c) Qualitatively, how do the corresponding energies of the ground and first excited states vary, as b goes from 0 to ∞? Sketch $E_1(b)$ and $E_2(b)$ on the same graph.
Hint: Recall the solutions to a single finite square well between $-a$ and a of depth V_0 are obtained from the equation

$$\tan\left(\frac{a}{\hbar}\sqrt{2m(E + V_0)}\right) = \sqrt{\frac{V_0}{E + V_0} - 1}.$$

What happens to the solutions to this equation when $z_0 = \frac{a}{\hbar}\sqrt{2mV_0}$ is large? It may help to first write the equation in terms of a unitless variable z and unitless constant z_0. Use this to make a qualitative statement about the behavior of the energies.

d) Suppose we use this potential as a 1-dimensional model for the potential energy experienced by an electron in a diatomic molecule. If the nuclei of the molecule are free to move, they will adopt the configuration of minimum energy. Use (b) to explain whether the electron pulls the nuclei together or pushes them apart.