Week 1 Worksheet 2

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Exercise 1. Translation. Let $\psi(x)$ be the position space wavefunction of a particle, and let $\varphi(p)$ be its wavefunction in momentum space.

a) Find the momentum space wavefunction of the position wavefunction $\psi(x+a)$ for some constant a. Make sure your answer is in terms of $\varphi(p)$.

b) Write down the Taylor series expansion of $\psi(x+a)$ about $a=0$, and show it's the same as

$$e^{ipa/\hbar}\psi(x).$$

Remark. We say that $e^{-ipa/\hbar}$ is the **translation operator** (by a).

Exercise 2. Probability Current. In class, you saw the quantity

$$J(x,t) = \frac{1}{2m}[\Psi^*(x,t)\,p\Psi(x,t) - \Psi(x,t)\,p\Psi^*(x,t)],$$

which is called **the probability current**.

a) Show, using the time-dependent Schrödinger equation, that

$$\frac{\partial J}{\partial x} + \frac{\partial \rho}{\partial t} = 0,$$

where $\rho(x,t) = |\Psi(x,t)|^2$ is the probability density. This equation is called the **continuity equation**.

b) Comment physically on what the continuity equation implies.
Hints: The 3D analog of the continuity equation is $\nabla \cdot \mathbf{J} + \partial\rho/\partial t = 0$. If you have a charged particle of charge q, and you multiply the continuity equation by q, what do you get?