Week 1 Worksheet
Math Review and de Broglie

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Exercise 1. Probability. Suppose you drop a match of length 1 cm onto a ruled sheet of paper whose lines are 1 cm apart. What is the probability that the match crosses a line on the paper?
Hints: First, figure out where the center of the match will land. Then, determine the angle of the match relative to the lines on the paper.

Exercise 2. de Broglie Formula. In this exercise, you will obtain the formula

$$\begin{aligned} p =& \hbar k \\ =& \frac{h}{\lambda}. \end{aligned}$$

a) The group velocity of a wave packet is defined as

$$\begin{aligned} v =& \frac{d\omega}{dk} \\ =& \frac{df}{d(1/\lambda)}. \end{aligned}$$

Differentiate the formula for the energy of a particle with frequency f, $E = hf$, to obtain a formula for $dE/d\lambda$ in terms of v and λ.

b) Using $E = \gamma m$ and $p = \gamma m v$, integrate your formula from (a) to get $p = h/\lambda$.

Hints: A trig substitution helps to evaluate the integral in (b), and $d(1/\lambda)$ can be evaluated using the chain rule. Recall that

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2}}$$

in natural units.

Exercise 3. de Broglie Wavelengths by Numbers. Calculate the de Broglie wavelength of the following particles.

a) A mass of 1 g moving with a velocity of 1 m/s.

b) A free electron with a kinetic energy of 200 eV.

c) A free electron with a kinetic energy of 50 GeV.

d) A free proton with a kinetic energy of 200 eV.

Hints: Use $hc = 1240$ eV nm, and note that in (c) you need to use the relativistic formula for momentum.