# Week 2 Worksheet 1 Solutions

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June 29, 2026

**Exercise 1. Warm up.** For these problems, you don't need to give rigorous proofs or all the details. Just make sure you understand the concepts.

a) Starting from the time-dependent Schrödinger equation (TDSE)

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi,$$

explain how to obtain the time-independent Schrödinger equation (TISE) $\hat{H}\psi = E\psi$. You don't have to go through the details of the derivation, but you do have to explain all of the steps.

b) Why is a general solution to the TISE a linear combination of definite energy solutions? *Hint*: If $D\psi = 0$ is a differential equation and $D$ is a *linear* differential operator, so that $D(a\psi_1 + b\psi_2) = aD\psi_1 + bD\psi_2$, then what does that tell you about linear combinations of solutions?

c) If we write down a linear combination $\psi(x) = \sum_i c_i \psi_i(x)$ as in (b), why is its time dependence given by $\Psi(x,t) = \sum_i c_i \Psi_i(x,t)$, where

$$\Psi_i(x,t) = e^{-iE_i t/\hbar}\psi_i(x)$$

is the solution to the TDSE given by the stationary state $\psi_i$ with energy $E_i$?

a) This is explained in Griffiths.

b) You can check for yourself that $H - E = D$ is a linear operator. Thus, solutions to the TISE with energy $E$ are linear combinations of such solutions.

c) Same as (b), but instead we check that the operator $H - i\hbar\partial/\partial t = D$ is linear. Then we know that solutions to the TDSE are just linear combinations of stationary state solutions.

**Exercise 2. Degeneracy.** In this exercise, you will show that same-energy solutions to the TISE that vanish anywhere on the $x$-axis, including $\pm\infty$, can never be degenerate. Suppose we have two solutions $\psi, \varphi$ to the time-independent Schrödinger equation with *the same* energy $E$.

a) Show that in this case

$$W = \varphi\psi' - \psi\varphi'$$

is a constant (independent of $x$). $W$ is called the **wronskian** of the two solutions. *Hint*: Show that $dW/dx = 0$.

b) Suppose that $\psi$ and $\varphi$ both vanish at some point $x$, including the possibilities $x \to \infty$ or $x \to -\infty$. Show that in this case

$$\psi(x) = c\varphi(x)$$

for some constant $c$.

a) Following the hint, we want to show $dW/dx = 0$. First, we compute this as

$$\frac{dW}{dx} = \varphi\psi'' - \psi\varphi'',$$

since the cross terms cancel. Since we have double derivatives of $\psi$ and $\varphi$ appearing, we can obtain a term like $\varphi\psi''$ by multiplying the TISE for $\psi$ by $\varphi$, and similarly for the term $\psi\varphi''$. Then, we subtract the two equations from each other to obtain

$$-\frac{\hbar^2}{2m}\frac{dW}{dx} = 0,$$

since the potentials and energies $E$ for both $\psi$ and $\varphi$ are the same. Thus, $W$ is a constant.

b) If $\psi$ and $\varphi$ both vanish somewhere, then we find that $W$ is in fact 0. Thus, we have a differential equation

$$\frac{\psi'}{\psi} = \frac{\varphi'}{\varphi}.$$

We can solve this equation to find $\ln(\psi) = \ln(\varphi) + \ln(c)$ to obtain the relation

$$\psi(x) = c\varphi(x).$$

**Bonus Exercise.** Consider a free particle solution to the TISE (a free particle means $V(x) \equiv 0$), and write a stationary state as

$$\psi_k(x) = \frac{1}{\sqrt{2\pi\hbar}}e^{ikx},$$

where $p = \hbar k$.

a) What is the energy $E_k$ of the stationary state $\psi_k$?

b) Is $\psi_k$ normalizable?

c) Is the most general free particle solution to the TISE a linear combination of stationary states, or something more complicated?
*Hint*: Recall that any function on a bounded interval, like $[0, 2\pi]$, (equivalently, any *periodic* function) can be expressed as a Fourier *series*. If the interval isn't bounded, then any function can still be expressed in this way, but now as a Fourier *integral*.

a) Since $p^2/2m = E$, we have

$$E_k = \frac{\hbar^2 k^2}{2m}.$$

b) $\psi_k$ isn't normalizable. In fact, its integral over the real line is undefined!

c) The most general solution is a Fourier integral

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int c_k e^{ikx} dx,$$

where the $c_k$ are coefficients. In fact, the $c_k$ assemble to give precisely the *function* $\varphi(k) = c_k$, where $\varphi(k)$ is the Fourier transform of $\psi(x)$.