Week 2 Worksheet 1

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Exercise 1. Warm up. For these problems, you don't need to give rigorous proofs or all the details. Just make sure you understand the concepts.

a) Starting from the time-dependent Schrödinger equation (TDSE)

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi,$$

explain how to obtain the time-independent Schrödinger equation (TISE) $\hat{H}\psi = E\psi$. You don't have to go through the details of the derivation, but you do have to explain all of the steps.

b) Why is a general solution to the TISE a linear combination of definite energy solutions?
Hint: If $D\psi = 0$ is a differential equation and D is a *linear* differential operator, so that $D(a\psi_1 + b\psi_2) = aD\psi_1 + bD\psi_2$, then what does that tell you about linear combinations of solutions?

c) If we write down a linear combination $\psi(x) = \sum_i c_i\psi_i(x)$ as in (b), why is its time dependence given by $\Psi(x,t) = \sum_i c_i\Psi_i(x,t)$, where

$$\Psi_i(x,t) = e^{-iE_it/\hbar}\psi_i(x)$$

is the solution to the TDSE given by the stationary state ψ_i with energy E_i?

Exercise 2. Degeneracy. In this exercise, you will show that same-energy solutions to the TISE that vanish anywhere on the x-axis, including $\pm\infty$, can never be degenerate. Suppose we have two solutions ψ, φ to the time-independent Schrödinger equation with *the same* energy E.

a) Show that in this case

$$W = \varphi\psi' - \psi\varphi'$$

is a constant (independent of x). W is called the **wronskian** of the two solutions.
Hint: Show that $dW/dx = 0$.

b) Suppose that ψ and φ both vanish at some point x, including the possibilities $x \to \infty$ or $x \to -\infty$. Show that in this case

$$\psi(x) = c\varphi(x)$$

for some constant c.

Bonus Exercise. Consider a free particle solution to the TISE (a free particle means $V(x) \equiv 0$), and write a stationary state as

$$\psi_k(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ikx},$$

where $p = \hbar k$.

a) What is the energy E_k of the stationary state ψ_k?

b) Is ψ_k normalizable?

c) Is the most general free particle solution to the TISE a linear combination of stationary states, or something more complicated?
Hint: Recall that any function on a bounded interval, like $[0, 2\pi]$, (equivalently, any *periodic* function) can be expressed as a Fourier *series*. If the interval isn't bounded, then any function can still be expressed in this way, but now as a Fourier *integral*.