Week 3 Worksheet 2

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July 8, 2026

Exercise 1. Warm up.

a) Sketch the solutions to the TISE for the potential $V(x) = \alpha\delta(x)$ in both cases $\alpha < 0$ and $\alpha > 0$.

b) What range of energies E are allowed solutions, and which correspond to bound states?

c) Do the same for the potential

$$V(x) = \begin{cases} V_0, & -a < x < a \\ 0, & x \notin [-a, a] \end{cases}$$

in both cases $V_0 < 0$ and $V_0 > 0$.

d) What range of energies E are allowed solutions in (c), and which correspond to bound states?

e) Explain what tunneling is and how it can arise in both (a) and (c) in the cases $\alpha, V_0 > 0$.

f) Explain how you would obtain the energies in (a) and (c).

Exercise 2. ***Griffiths 2.47*****.** Consider a double finite square well:

$$V(x) = \begin{cases} -V_0, & x \in (-a - b/2, b/2) \\ -V_0, & x \in (b/2, a + b/2) \\ 0, & \text{otherwise} \end{cases}.$$

a) Sketch this potential.

b) Sketch the allowed wave functions for the cases (i) $b = 0$, (ii) $b \approx a$, and (iii) $b \gg a$. Include also the ground state and first excited state.
Hint:

c) Qualitatively, how do the corresponding energies of the ground and first excited states vary, as b goes from 0 to ∞? Sketch $E_1(b)$ and $E_2(b)$ on the same graph.
Hint: Recall the solutions to a single finite square well between $-a$ and a of depth V_0 are obtained from the equation

$$\tan\left(\frac{a}{\hbar}\sqrt{2m(E + V_0)}\right) = \sqrt{\frac{V_0}{E + V_0} - 1}.$$

What happens to the solutions to this equation when $z_0 = \frac{a}{\hbar}\sqrt{2mV_0}$ is large? It may help to first write the equation in terms of a unitless variable z and unitless constant z_0. Use this to make a qualitative statement about the behavior of the energies.

d) Suppose we use this potential as a 1-dimensional model for the potential energy experienced by an electron in a diatomic molecule. If the nuclei of the molecule are free to move, they will adopt the configuration of minimum energy. Use (b) to explain whether the electron pulls the nuclei together or pushes them apart.