Week 5 Worksheet 1

Jacob Erlikhman

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Exercise 1. On the homework, you will show that the matrix elements of x and p in the harmonic oscillator eigenbasis $\{|n\rangle\}$ are given by

$$x_{mn} = \langle m|x|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}(\delta_{m,n+1}\sqrt{n+1} + \delta_{m,n-1}\sqrt{n})$$
$$p_{mn} = i\sqrt{\frac{m\omega\hbar}{2}}(\delta_{m,n+1}\sqrt{n+1} - \delta_{m,n-1}\sqrt{n}).$$

a) For finite-dimensional matrices, show that $\text{tr}([A, B]) = 0$, where for an $(n \times n)$-matrix A, the **trace** $\text{tr}(A)$ is the sum of the diagonal entries of A.
Hint: Write $\text{tr}(AB)$ in summation notation as $\sum_{i,j} c_{ij}$; that is, determine the scalars c_{ij} in terms of the matrix elements of A and B.

b) *Paradox.* From the result of (a), it would seem that the matrices which you will compute on the homework imply that $\hbar = 0$, since $[x, p] = i\hbar$. Use the matrix elements x_{mn} and p_{mn} to calculate xp and px, and explain in detail why the paradoxical conclusion $\hbar = 0$ is invalid.