Week 7 Worksheet 1

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Exercise 1. On midterm 1, you showed that if $T(a) = e^{-ipa/\hbar}$, then $T(a)\psi(x) = \psi(x-a)$. Generalize this result to three-dimensions.

Exercise 2. For R a rotation matrix in 3-dimensions, let $U(R)$ be the operator defined by $U(R)\,|\mathbf{x}\rangle = |R\mathbf{x}\rangle$.

a) Show that $U = U(R)$ is unitary.
Hint: First, show that $\langle \mathbf{x}'|U^\dagger U|\mathbf{x}\rangle = \langle \mathbf{x}'|\mathbf{x}\rangle$ for all $\mathbf{x}$ and $\mathbf{x}'$. What else is left to show that $U(R)$ is unitary?

b) Show that $U(R_1)U(R_2) = U(R_1 R_2)$.

c) Show that $U(I) = \mathbb{1}$, where I is the identity (3×3)-matrix.

d) Show that $U(R^{-1}) = U(R)^{-1}$.

e) Show that if

$$|\psi'\rangle = U(R)\,|\psi\rangle\,,$$

then

$$\psi'(\mathbf{x}) = \psi(R^{-1}\mathbf{x}).$$

Hint: Use a resolution of the identity.

Remark. This result is analogous to the result of the previous exercise,

$$T(\mathbf{a})\psi(\mathbf{x}) = \psi(\mathbf{x} - \mathbf{a}).$$