

# Week 14 Worksheet

## Bouncing Ball

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**Exercise 1. Griffiths 9.6 Quantum Bouncing Ball.** A ball of mass  $m$  bounces elastically on the floor.

- a) What is the potential as a function of the height  $x$  above the floor?

*Hint:* If the ball can't go below  $x = 0$ , what is the potential in that region?

- b) Solve the Schrödinger equation. You don't need to normalize your solution.

*Hint:* You should get Airy's differential equation,  $\psi''(z) - z\psi(z) = 0$ . One way to manipulate the Schrödinger equation into such a form is to notice that for  $\psi''(x) - \alpha^3 x \psi(x) = 0$ , the substitution  $z = \alpha x$  does the trick. The solutions of this equation are the Airy functions,  $\text{Ai}(z)$  and  $\text{Bi}(z)$ . The graphs of these functions are below.

- c) Calculate (approximately) the first 4 energies, using  $g = 10 \text{ m/s}^2$  and  $m = 0.100 \text{ kg}$ .

*Hint:*

$$\left(\frac{mg^2\hbar^2}{2}\right)^{1/3} \approx 5 \cdot 10^7 \text{ eV} = 8 \cdot 10^{-12} \text{ J}.$$

- d) Now, analyze this problem using the WKB approximation. Find the allowed energies  $E_n$  in terms of  $m$ ,  $g$ , and  $\hbar$ .

*Hint:* The connecting WKB wavefunctions are

$$\psi(x) = \begin{cases} \frac{2D}{\sqrt{p(x)}} \sin\left(\frac{1}{\hbar} \int_x^{x_2} p(x') dx' + \frac{\pi}{4}\right), & x < x_2 \\ \frac{D}{\sqrt{p(x)}} \exp\left(-\frac{1}{\hbar} \int_{x_2}^x |p(x')| dx'\right), & x > x_2 \end{cases}.$$

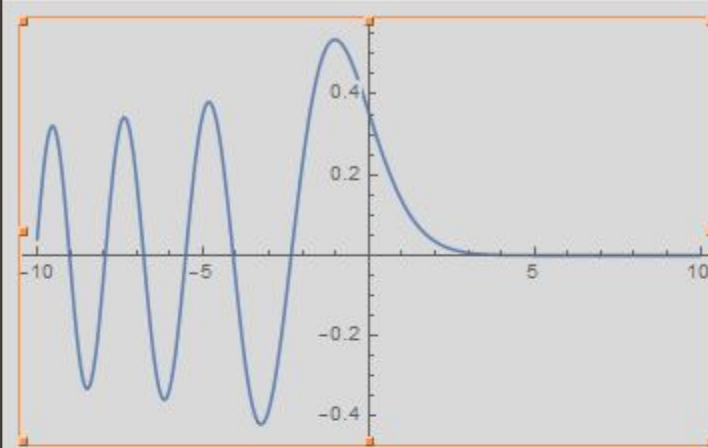
- e) Plug in the values from (c), and compare the WKB calculation to the exact one for the first four energies.

*Hint:*

$$\frac{(3m^2 g \pi \hbar)^{2/3}}{2m} \approx 6.6 \cdot 10^{-4} \text{ eV} = 1 \cdot 10^{-22} \text{ J}$$

- f) How large would  $n$  have to be to give the ball an average height of 1 meter above the ground?

```
Plot[AiryAi[z], {z, -10, 10}]
```



```
Plot[AiryBi[z], {z, -10, 10},  
PlotRange -> {{-10, 10}, {-0.5, 2}}]
```

