Week 5 Worksheet
More Perturbation Theory

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September 26, 2025

Exercise 1. ***Griffiths 7.54.*** Last week, you derived the first order correction to the expectation value of an observable A in the n^{th} energy eigenstate of a system perturbed by H^1. You found

$$\langle A\rangle^1 = 2\text{Re}\sum_{m\neq n}\frac{\left\langle n^0\middle|A\middle|m^0\right\rangle\left\langle m^0\middle|H^1\middle|n^0\right\rangle}{E_n^0 - E_m^0}.$$

Suppose we have a particle of charge q in a weak electric field $\mathbf{E} = E_{\text{ext}}\hat{x}$, so that $H^1 = -qE_{\text{ext}}x$. This induces a dipole moment $p_e = qx$ in the "atom." The expectation value of p_e is proportional to the applied field, and the proportionality factor is called the **polarizability**, α. Show that

$$\alpha = -2q^2\sum_{m\neq n}\frac{|\langle n^0|x|m^0\rangle|^2}{E_n^0 - E_m^0}.$$

Find α for the ground state of a 1-D harmonic oscillator, and compare the classical answer.

Hint: Recall that x can be written in terms of creation and annihilation operators. Given

$$H^0 = \frac{1}{2m}\left[p^2 + (m\omega x)^2\right],$$

you can derive what a and $a^\dagger$ should be in terms of x and p by using the sum of squares formula. To get the "usual" form, rescale each of them by $a \to \frac{1}{\sqrt{\hbar\omega}}a$ (so that the hamiltonian can be written $\frac{H^0}{\hbar\omega} = a^\dagger a + 1/2$).

Exercise 2. ***Griffiths 7.45.*** **Stark Effect in Hydrogen.** When an atom is placed in a uniform electric field $\mathbf{E}_{\text{ext}}$, the energy levels are shifted. This is known as the **Stark effect**. You'll analyze the Stark effect for the $n = 1$ and $n = 2$ states of hydrogen. Suppose $\mathbf{E}_{\text{ext}} = E_{\text{ext}}\hat{z}$, so that

$$H^1 = eE_{\text{ext}}r\cos\theta$$

is the perturbation of the hamiltonian for the electron, where $H^0 = \frac{p^2}{2m} - \frac{e^2}{r}$.

a) Show that the ground state energy is unchanged at first order.

b) How much degeneracy does the first excited state have? List the degenerate states.

c) Determine the first-order corrections to the energy. Into how many levels does E_2 split?

Hint: All W_{ij} are 0 except for two, and you can avoid doing all of the zero integrals in this problem by using symmetry and selection rules. Since selection rules weren't covered in this course, the specific one you'll need is that

$$\langle n\ell' m' | n\ell m\rangle = 0$$

unless $\ell + \ell'$ is odd. You'll also need the following

$$\begin{aligned}\psi_{210} &= \frac{1}{2\sqrt{6}} a^{-3/2} \frac{r}{a} e^{-r/2a} \sqrt{\frac{3}{4\pi}} \cos\theta \\ \psi_{200} &= \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \frac{1}{2\sqrt{\pi}}.\end{aligned}$$

d) What are the "good" wavefunctions for (b)? Find the expectation value of the electric dipole moment in each of these states.