Week 4 Worksheet
Free Electron Gas

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Exercise 1. Suppose you have N electrons in a box of side length L.

  a) Show that the Fermi energy is

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3N\pi^2}{L^3} \right)^{2/3}.$$

  b) Find the total energy of the electrons in terms of E_F.

Exercise 2. Now, consider a free electron gas in two dimensions, confined to a square of side length L.

  a) *Griffiths5.30.* Find the Fermi energy in terms of N and L, and show that the average energy of the particles is $E_F/2$.

  b) Let $g(E)\,\mathrm{d}E$ be the number of particles with energy E in the interval $\mathrm{d}E$. $g(E)$ is called the **density of states** and is useful in various problems in quantum statistical mechanics. Calculate $g(E)$ for the particles. Your formula should be constant, i.e. independent of E.