

# Review Session Problems 2

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**Exercise 1.** When we solve the hydrogen atom, we assume that the nucleus is a point charge. In this problem, we will compute the approximate change to the energy levels due to the finite size of the nucleus. This is called the **volume effect**. Model the nucleus as a uniform sphere of radius  $r_0 A^{1/3}$ , where  $A^{1/3}$  is the number of nucleons (so this works for e.g. deuterium) and  $r_0 = 1.3 \cdot 10^{-13}$  cm.

a) What is the potential  $V(r)$ ?

*Hint:* Outside the nucleus,  $V(r)$  is just the Coulomb potential. Inside the nucleus, use Gauss' law to determine  $V(r)$ .

b) What is  $H'$ , where  $H^0$  is the hydrogen atom hamiltonian?

c) It turns out that the  $\ell = 0$  states are the most affected by this perturbation, with the  $\ell > 0$  states only very slightly affected. Calculate the correction to the energy levels for all states with  $\ell = 0$ . Note that

$$R_{n0}(0) = \frac{2}{(na_0)^{3/2}},$$

where  $a_0 = \hbar^2/m_e^2$ .

d) For hydrogen, calculate the correction to the  $n = 1$  and  $n = 2$  states in eV.

e) Fine structure is of order  $\alpha^4 mc^2$ . Compare the magnitude of the volume effect to that of fine structure.

**Exercise 2.** Explain the physical origins of

a) fine structure

b) Lamb shift

c) hyperfine structure.

**Exercise 3.** *Griffiths 8.19* Find the lowest bound on the ground state of hydrogen using the variational principle and an exponential trial wavefunction,

$$\psi(\mathbf{r}) = Ae^{-br^2},$$

where  $A$  is determined by normalization and  $b$  is a variational parameter. Express your answer in eV.

**Exercise 4.** *Griffiths 9.18* When we turn on an external electric field, it should be possible to ionize the electron in an atom. A crude model for this is to suppose that a particle is in a very deep, one-dimensional finite square well.

- a) What is the energy of the ground state, measured up from the bottom of the well? Assume that  $V_0 \gg \hbar^2/ma^2$ .
- b) Introduce the perturbation  $H' = -\alpha x$ , where  $\alpha \equiv eE_{\text{ext}}$ . Assume that  $\alpha a \ll \hbar^2/ma^2$ , and sketch the total potential, noting that the electron can tunnel out in the direction of positive  $x$ .
- c) Calculate

$$\gamma = \frac{1}{\hbar} \int |p(x)| dx,$$

and estimate the time it would take for the particle to escape,

$$\tau = \frac{2x_1}{v} e^{2\gamma},$$

where  $x_1$  is the distance the electron must travel to reach the tipping point of the potential and  $v$  is the speed of the electron.

- d) Plug in some numbers, e.g.  $V_0 = 20$  eV,  $E_{\text{ext}} = 7 \cdot 10^6$  V/m,  $a = 10^{-10}$  m. Calculate  $\tau$ , and compare it to the age of the universe.