Week 2 Worksheet Solutions
Special Relativity (Possibly Review)

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Exercise 1. Squishing a Bug. A rectangular piece of wood is cut from a board. A T-shaped cut is then made in this piece to obtain two pieces, one U-shaped and one T-shaped, see Fig. 1 below. The length of the protruding piece is D, and furthermore a bit of the hole is cut out, so that it is of depth $D + \varepsilon$, where $\varepsilon \ll D$. When the pieces are put together, a bug of a size less than ε is able to live between the two pieces. Now, the T-shaped piece is accelerated to a relativistic velocity toward the second piece. They collide. Does the bug get squashed?

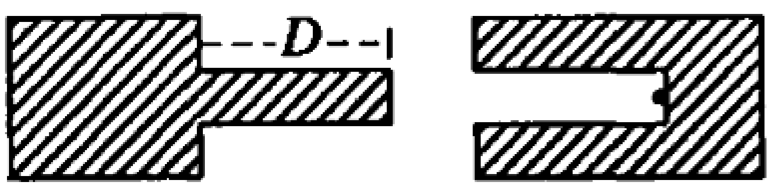

Figure 1: Two blocks and a bug in their rest frame.

In the frame of the T-shaped piece, it is clear that the bug gets squashed, since the hole gets smaller, while the protruding piece of length D is unchanged. So the question is what happens in the frame of the bug. The protruding piece is now of length less than D; however, as the sides of the T hit the sides of the U, the front of the protruding piece *does not yet know that they have hit*. This is because any information of the collision can be communicated at a speed not greater than the speed of light (but in fact at a speed closer to the speed of sound). Thus, the front of the protruding piece will continue moving until it squashes the bug.

Exercise 2.

a) A meterstick is moving in a straight horizontal line (parallel to the orientation of the stick). A metal plate with a 1 m diameter circular hole in it is rising vertically, perpendicular to the stick. The stick is moving so that its contracted length is 10 cm, so in our frame it easily fits through the hole and ends up on the other side of the plate. Explain what happens in the reference frame of an observer sitting on the stick. See Fig. 2a.

b) A 1 m hole is cut out of a thin table. A thin meterstick moves along the table toward the hole. Explain what happens in both reference frames. See Fig. 2b.

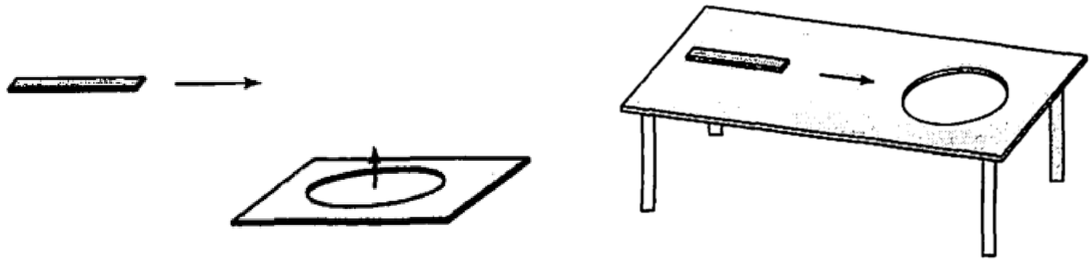

Figure 2: Exercise 2(a) left and 2(b) right.

a) Let's consider what happens in the "easy" frame, i.e. the one in which both the stick and plate are moving. In this case, the time when the front of the hole and the back of the hole reach the stick are *the same*, and the stick easily fits through the hole. Both the hole and the plate continue on their journeys. Now, in the frame of an observer who is sitting on the stick, the time when the front and back of the hole reach the plane of the stick is *different*! Thus, the hole appears to be tilted to the stick. Indeed, we can place a clock on the front and back of the hole (for me, the back is the one which is closer to the stick). Since leading clocks lag, this clock will be slow compared to the one at the front, so it will reach the stick at a later time than the front one. The only way this can occur is if the entire plate is tilted in the frame of an observer on the stick!

b) In the frame of an observer on the table, the meterstick is much shorter and then easily fits through the hole. The stick will continue straight until its center of mass reaches the side of the hole, at which point it will begin falling into the hole and end up falling completely through. On the other hand, in the frame of the meterstick, the hole is approaching it and appears to be a narrow oval. However, the part of the stick which is *behind* the center of mass will only know that the front is falling after a time of $\Delta x/c$, where Δx is the distance between the falling part and the back part. Due to information being able to move no faster than the speed of light, rather than a rigid stick, we should instead think of the stick as a chain, where each piece of the chain is rigid. In particular, this means that in the frame of an observer sitting on the stick, the stick will *curve* or "dive" into the hole: The back part will still be sliding on the table while the front half is falling in! This will allow it to fit through the hole even though the hole is (much) smaller than the length of the stick.

Exercise 3. Superball Clocks. Consider a "clock" which is made up of a ball bouncing back-and-forth between two walls in a gravity-free space. The walls are a distance D apart, and the ball moves with constant speed v_0. An identical clock in a spaceship moves past us to the right at speed V. The motion of the ball in our frame is perpendicular to the velocity of the ship $\mathbf{V}$.

a) Draw a picture of the clocks in i) their rest frames and ii) the frames in which they're moving, indicating the ball's trajectory in each.

b) How long does it take the ship clock to tick once?

c) Does the ship clock run slow, fast, or at the same rate as our clock, measured in our frame? Explain. Does the result agree with time-dilation formulas or not?

a) See Fig. 3 below.

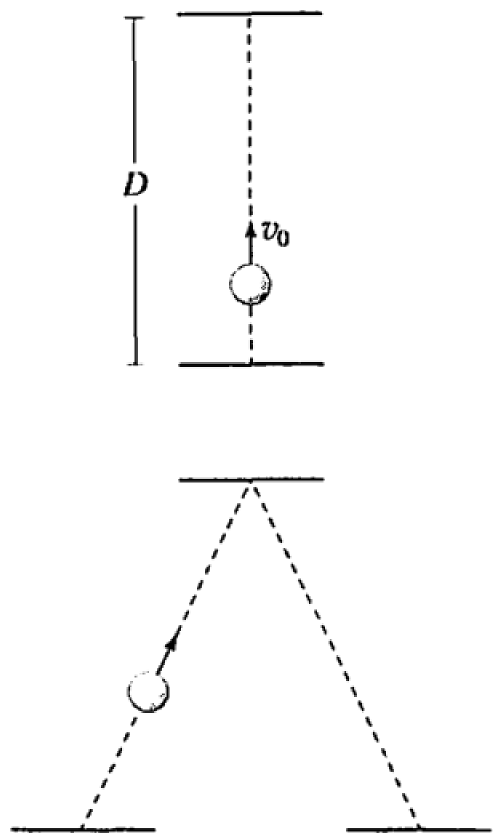

Figure 3: Diagram of the ball clock on the ship in the ship frame (top) and the lab frame (bottom).

b) In the ship frame, the clock takes a time $T = 2D/v_0$ to make one cycle. Let **V** be along the x-axis and the ball's motion along the y-axis. To calculate the time that this takes in our frame, we must use Lorentz transformations. In particular, we calculate

$$T' = \gamma(T - VX),$$

where X is the coordinate along the x-axis in the ship frame at which the ball returns to its original position. Of course, $X = 0$, so

$$T' = \gamma T.$$

Now, we must be careful to calculate what happens in our frame. Here, the ball travels a distance $X' = 2\gamma VD/v_0$ during its path from beginning to end; thus, the elapsed time in the ship frame is

$$\begin{aligned} T =& 2\gamma(\gamma D/v_0 - \gamma V^2 D/v_0) \\ =& 2D/v_0, \end{aligned}$$

as it should be. So everything is consistent. Note that *in our frame* the ship's second is γ times *longer* than our second! This is not a problem, because the amount of time that passes on the ship (i.e. *in the ship frame*) in one of our seconds is γ times less than our second.