Week 3 Worksheet Light Cones and Spacetime Diagrams

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Exercise 0. Warm Up.

- a) A chicken crosses the road. Is the spacetime interval between the beginning and end of its journey spacelike, timelike, or null?
- b) Close your eyes and then open them briefly. What set of points in spacetime do you see during that brief time interval?

Exercise 1. The sun explodes.

- a) We sit down to eat lunch (on Earth) four minutes after the explosion. Is the interval between these two events spacelike or timelike? If it is spacelike, find the reference frame (i.e. find the speed of this frame relative to the Sun-Earth rest frame) in which the two events are simultaneous. If it is timelike, find the reference frame in which the two events occur at the same location in space (and happen sequentially).
- b) Repeat part (a) if we sit down to eat lunch 10 minutes after the explosion.
- **Exercise 2. Spacetime Shootout.** Spaceships B and C, starting at the same location when each of their clocks reads zero, depart from one another with relative velocity $\frac{3}{5}c$. One week later according to B's clocks, B's captain goes berserk and fires a photon torpedo (which travels at the speed of light) at C. Similarly, when the clocks on C read one week, C's captain does the same at B.
 - a) Draw a two-dimensional spacetime diagram of the events in B's frame.
 - b) Same as (a) but for C's frame.
 - c) Which ship gets hit first? Is there a paradox?

Exercise 3. Time Travel. Repeat Exercise 2, but this time, instead of a photon torpedo, B's captain sends a tachyonic message *at infinite speed* to C (and C's captain does nothing). Draw both spacetime diagrams including the tachyon paths and lightcones (centered on the point of departure). Comment on the shape of the tachyon path in C's frame.