Week 4 Worksheet
4-vectors, Energy-Momentum, and the Metric

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Exercise 1.

a) Show that the squared length of any particle's four-velocity is -1.

b) Show that

$$m^2 = E^2 - p^2$$

in natural units (i.e. in units such that $c = 1$).
Hint: The p^2 in this formula is *not* the squared length of the 4-momentum p; rather, it is the squared length of the 3-vector $\mathbf{p}$!

Exercise 2. A particle of rest mass m and 4-momentum p is examined by an observer with 4-velocity u. Show the following statements.

a) The energy the observer measures is

$$E = -p \cdot u.$$

Hints: First, check that this formula holds if the observer is in the particle's own rest frame. Then argue that this implies that it is true in all frames. You can also check this explicitly in coordinates.

b) The rest mass the observer attributes to the particle is

$$m^2 = -p^2.$$

c) The 3-momentum the observer measures has magnitude

$$|\mathbf{p}'| = \sqrt{(p \cdot u)^2 + p^2}.$$

d) The 3-velocity $\mathbf{v}$ the observer measures has magnitude

$$|\mathbf{v}| = \frac{|\mathbf{p}'|}{E}.$$

e) The 4-vector v ("ordinary velocity"), whose components in the observer's frame are

$$v^0 = 0, \quad v^j = dx^j/dt,$$

is given by

$$v = \frac{p + (p \cdot u)u}{-p \cdot u}.$$

Here, t is understood to be the time *for the particle*.