Week 6 Worksheet
Equivalence Principle and Some Geometry

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Exercise 0. Warm up. In solving the following short problems, make sure to give a physical explanation of your solutions *without using the equivalence principle*!

a) While sitting in a car at a stoplight, you're holding a helium-filled balloon on a string. The light turns green and the car accelerates. What happens to the balloon?

b) A balloon filled with helium is released at rest in the middle of a drifting spaceship in gravity-free space. The spaceship is filled with air. What does the balloon do?

c) The spaceship accelerates forward. Does the balloon drift to the front or back of the rocket?

d) You are holding a cup of hot tea in a coasting car. The driver steps on the brakes. Which way should you tip the cup so its contents won't spill out?

e) A car is at rest with one of its doors open. Describe how to close the door by accelerating the car.

Exercise 1. Find the altitude of a satellite in a circular orbit about the Earth whose clocks run *at the same rate* as Earth clocks. The radius of the Earth is 6.4 km.
Hints: You can determine the value of GM_E by noting that $g = GM_E/R_E$. Also recall that the ratio of the rate signals are received at A to the rate signals are emitted at B is

$$1 + \Phi_A - \Phi_B.$$

Exercise 2. Life on a Cylinder. Consider a cylinder of radius r in ordinary euclidean 3-space. Write down the metric on the cylinder induced from the euclidean one on 3-space as follows.

a) Model the cylinder as $Z = S^1_r \times \mathbb{R}$, where S^1_r denotes the circle of radius r. There is a map from the cylinder minus a line to $\mathbb{R}^2$, $Z \smallsetminus \ell \to \mathbb{R}^2$, where we cut out a line ℓ along the "vertical" direction of the cylinder and then "roll" the cylinder out onto the plane. Argue that this map is an **isometry**, so that it gives an equivalence between the metric on the cylinder minus a line and the metric on $\mathbb{R}^2$.

b) Part (a) tells you that the shortest path between two points will be one which subtends an angle which is not greater than π. Given any two points, we can now choose which line to cut out of the cylinder to determine the distance between them. Using this fact and the isometry in (a), compute the metric.